

YHQL YLGL YLFL — J. Caesar

Cryptography

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The Menu

• Symmetric Crypto





The Menu

- Symmetric Crypto
- Asymmetric Crypto (aka Public-Key)





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- Symmetric Crypto
- Asymmetric Crypto (aka Public-Key)
- Hashes, MICs, and MACs





Cryptography _

	Aliceの偏光	Eveの 基底		
	ICHER T	EUD	間違い	
	8050 正しい 基底 間違い	ビットは作られ	違うビット外的ペイ270日 よい	101
Evebili	調違った基底を通	遅ぶ確率(よ	1	
さらに、	BobbiAliceと違	う偏光を測定	2 とまる確率が 1	
よって日	iveの盗聴が感知	はれる確率	1 2 1 (1) ^N	
NEON	照合して、盗聴が	が感知されな		
10ビット	照合する場合、	盗聴がばれ		= 5.6×10 ²
100ビッ	ト照合する場合	(2(t		
1000	ット昭合する場合	ta =		
			ビーム遮断では盗い	徳がばれる



Terminology _____

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Avoid subscript k; easily confused with subscript K.

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- Examples: RSA, Elgamal, ECC



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With AES, you can choose m and n independently from $\{128, 160, 192, 224, 256\}$.







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Properties Of a Good Block Cipher

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That's a bit difficult to attain in practice, because we can't see into the future!

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Electronic Codebook Mode (ECB)







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Identical plaintext blocks lead to identical ciphertext blocks. This makes it possible to find all employees with the same salary as employee X...

... without breaking the encryption scheme.

Cipher Block Chaining (CBC)



The "IV" is a random initialization vector that is sent unencrypted with the message.



Features of CBC

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In most cases, security is not weakened by choosing a constant IV for each message, but there are exceptions (see exercises).



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Assume the plaintext is "Trudy____R&D____\$20000____"

The character 2 has the bit representation 00110010. 3 is 00110011. Can Trudy force this single bit to change?



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If Trudy flips the last bit of C_1 , block 1 will decrypt as garbage, but C_2 will decrypt as $\mathbb{R}_2 \oplus 1 = \mathbb{R}_2 \oplus 1 = \mathbb{R}_3$, a 50% increase in Trudy's salary!



Problems With CBC (2) _

In CBC, $p_i = c_{i-1} \oplus D_K(c_i)$ where c_0 is the IV. Hence, $D(c_i) = c_{i-1} \oplus p_i$.





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Arrangement	Decryption
$c_0 c_1 c_2 c_3$	$ p_1 p_2 p_3$
$c_1 c_0 c_2 c_3$	$c_1 \oplus D(c_0) c_0 \oplus D(c_1) p_3$
$c_0 c_1 c_2 c_2$	$p_1 p_2 c_2\oplus D(c_2)=p_3\oplus D(c_3)\oplus D(c_2)$

It is improbable that rearranged messages will decrypt to something useful, but it's still a threat.

Feedback Modes (CFB, OFB)



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Feedback Modes Explained



OFB and CFB generate a *one-time pad* consisting of pseudo-random numbers from an IV and a key: $c_i = p_i \oplus k_i$, where k_i is the key stream generated by the IV and K.



Feedback Modes Explained



OFB	CFB
Uses only key and IV to ge-	Also uses message
nerate key stream	
Encryption pad can be com-	Must wait for plaintext
puted beforehand	
Can generate ciphertext as	Can generate ciphertext as
fast as the plaintext appears	fast as plaintext appears if
	block sizes match



Effect of Transmission Errors and Attacks



Error	OFB Decryption	CFB Decryption
Garbled bits	Garbles rest of mes-	Garbles only these
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Since $p_i = c_i \oplus k_i$, we must substitute $p'_i \oplus k_i$ for c_i if we want the *i*-th ciphertext character to decrypt to p'_i .





Counter Mode (CTR)



Key stream can again be precomputed (like OFB) and decryption can start at any point (not just at the beginning).

Advice



Encrypt What	Recommendation
Files	CBC with a random IV (especially
	if you want to access the file non-
	sequentially). Also use a good Messa-
	ge Integrity Code (MIC) in order to de-
	tect modification of the ciphertext.
Net Sessions	CFB or OFB with a random IV or native
	stream cipher like RC4. Protect each
	packet with a MIC.
Short Database Fields	CBC with random IV and MIC.
Encryption Keys	ECB with MIC.





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Do not deploy any algorithm without checking whether it has been broken in the meantime. It happens.

More Advice on Algorithms

Do not use these ciphers; they are broken: GDES, DESX, (and most other DES variants), Bass-O-Matic, Khufu, Khafre, FEAL, Akelarre, SPEED, Enigma 2000, JEL, StreamBuddy, and many *many* more.



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N.B.: DES is an excellent cipher; it has withstood about 30 years of cryptanalysis. The best way of attacking DES is brute force. The problem with DES is that brute force is too easy.



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Why Isn't He Showing Source Code?

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Example: I've seen an application that fed the plaintext back instead of the ciphertext, turning CFB into "PFB", which exposes patterns in the input. (Code change: one identifier.)



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 {[M']_{Alice}}_{Alice} = M'.
- It is not necessary that $\{M\}_{Alice}$ be in the domain of $[\cdot]_{Alice}$. (Signature without encryption.)

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There are crypto libraries out there that are so orthogonal that they allow you to specify RSA with CBC, *but that's nonsense!*

It's even more important than in the case with symmetric crypto *not to write your own RSA package*, because there are even more things that can go wrong when you don't do it right.





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Let p and q be two different odd primes. Let n = pq. We have $\phi(n) = (p-1)(q-1)$. Choose e such that gcd(e, p-1) = 1 and gcd(e, q-1) = 1. Note that this means that $gcd(e, \phi(n)) = 1$.





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Let p and q be two different odd primes. Let n = pq. We have $\phi(n) = (p-1)(q-1)$. Choose e such that gcd(e, p-1) = 1 and gcd(e, q-1) = 1. Note that this means that $gcd(e, \phi(n)) = 1$.

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Some choices of *p* and *q* are better than others! Beware!

To encrypt a message 0 < P < n, compute $C = P^e \mod n$. To decrypt a message, compute $P' = C^d \mod n$.





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When *P* is a multiple of *p* or *q*, things also work out. (Having P = kp would expose *p*, because $gcd(P^e \mod n, n) = p$, but that is just as likely as correctly guessing *p* or *q*.)





RSA Pitfalls: Small Encryption Exponent

You want to send a message P to three participants with public keys $(3, n_1)$, $(3, n_2)$, and $(3, n_3)$. Encryption is:

$$C_j = P^3 \mod n_j$$
 for $1 \le j \le 3$.

By the Chinese Remainder Theorem, we can compute some x with $C_j = x \mod n_j$ ($1 \le j \le 3$), if the n_j are pairwise relatively prime (very likely).

This x is unique modulo $n_1n_2n_3$. We compute the smallest nonnegative such x.

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```
Solution: Choose e = 65537.
```



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Most messages are indeed small (112-bit or 128-bit encryption keys, for example), where there's a chance that this will happen.



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Never roll your own RSA routines!

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All but the last requirements are also required of hash functions.





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Privacy And Integrity (1)

Can we get encryption *and* integrity protection at the same time?



Privacy And Integrity (2)



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Privacy And Integrity (3)



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The Moral _



You might be able to get integrity and privacy protection in one pass over the data, but how to do that is still under active research.



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Your best best will be to do two passes over the data; the first pass should compute a hash (or keyed hash; later), and the second pass should encrypt.





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Do not try to take shortcuts in crypto!



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- Given a message, it is infeasible to find another message with the same checksum.

Note that it cannot be *impossible* to find collisions, because of the pigeonhole principle: If you have infinitely many messages, but only finitely many hashes, some messages must hash to the same value.



How Infeasible is Finding a Collision?

Let's say the hash function is cryptographically strong, but I still want to crack it. I follow the following algorithm:

1. Set $S \leftarrow \emptyset$.

- 2. Generate a new, random message m and its hash h(m).
- 3. If $(m, h(m)) \in S$, terminate the algorithm. Otherwise, set $S \leftarrow S \cup (m, h(m))$ and repeat step 2.

How often will step 2 have to be executed before the algorithm terminates? (We may assume that the messages that are generated contain no duplicates.)

Collision Probability (1) _

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$$P(k) = \frac{N}{N} \cdot \frac{N-1}{N} \cdot \cdot \cdot \frac{N-k+1}{N} = \prod_{j=0}^{k-1} \left(1 - \frac{j}{N}\right)$$

Now we want to know the first k for which P(k) < 0.5.



Collision Probability (2)

$$\begin{split} \prod_{j=0}^{k-1} \left(1 - \frac{j}{N}\right) &< \left(\frac{1}{k} \sum_{j=0}^{k-1} \left(1 - \frac{j}{N}\right)\right)^k \\ &= \left(1 - \frac{k-1}{2N}\right)^k \\ &\approx \left(1 - \frac{k}{2N}\right)^k \\ &< \exp(-k^2/2N). \end{split}$$

To find *k* for which P(k) < 0.5, we solve $\exp(-k^2/2N) < 0.5$ for *k* to yield $k > \lambda \sqrt{N}$ where $\lambda = \sqrt{2 \ln 2} \approx 1.18$.

If $N = 2^n$, and if n is even, $\sqrt{N} = 2^{n/2}$. We'll leave out the factor of λ (since it's so close to 1).



Collision Probability (3)

For an *n*-bit hash, we have to hash about $2^{n/2}$ messages before we can expect a collision with probability at least 1/2.

That means that





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Any hash function that has less than 128 bits of hash should be considered insecure and weak *and should not be used*.

Well-Known Hash Functions

For some reason, it seems to be easier to create good hash functions than to create good encryption schemes. Some good hash functions are:

Name	Bits	Comment
MD5	128	Less fast than predecessor MD4 (*)
SHA-1	160	Standard (*)
RIPEMD-160	160	

(*) Length limited to be less than 2^{64} bits; but "If you can't say something in 2^{64} bits, you probably shouldn't say it at all".

If we could hash one Terabyte per second (which we can't), hashing the entire 2^{64} bits would take about 550,000 years to compute.



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How can we add a key to the message digest algorithm?



MACs With Hashes And Keys (1) $_{-}$

Alice and Bob agree on a shared secret K_{AB} . If Alice sends a message m to Bob, she concatenates K_{AB} and m and sends $hash(K_{AB}|m)$ as the MAC.





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The key to the attack is that it's possible to compute hash(x|y) if you know hash(x) and y.

That means that if Eve sees $hash(K_{AB}|m)$, she can compute

 $hash(K_{AB}|m|Romeo must die)$





MACs With Hashes And Keys (2) $_$

Solution: HMAC, which is becoming the standard MAC.



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MACs With Hashes And Keys (2)

Solution: HMAC, which is becoming the standard MAC. HMAC is provably "secure" if the underlying hash algorithm is

• It has collision resistance; and

"secure":

• if the attacker doesn't know the key K, he cannot compute MAC(K, x) even if he sees arbitrarly many MAC(K, y) values.



HMAC



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Libraries: OpenSSL and cryptlib (1) _____

	OpenSSL	cryptlib
Author	Eric Young, OpenSSL	Peter Gutmann
	Project Team	
Since	1990's	1990's
Vuln's	several	none
Scope	wide, many OSS pro-	wide, mostly non-OSS
	jects	projects
Approach	bunch of functions	application support
Runs on	mostly Unix and Win-	tons of stuff: mainfra-
	dows	mes to embedded sy-
		stems
License	OSS	OSS
Free?	all use	noncommercial use



Libraries: OpenSSL and cryptlib (2)_

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Once it's set up, encrypting an email message is a matter of three lines, including S/MIME enveloping.



Summary

• Symmetric Crypto





Summary

- Symmetric Crypto
- Asymmetric Crypto (aka Public-Key)





Summary

- Symmetric Crypto
- Asymmetric Crypto (aka Public-Key)
- Hashes, MICs, and MACs





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